

# Is Sense-Making at the Core of Your Classroom?

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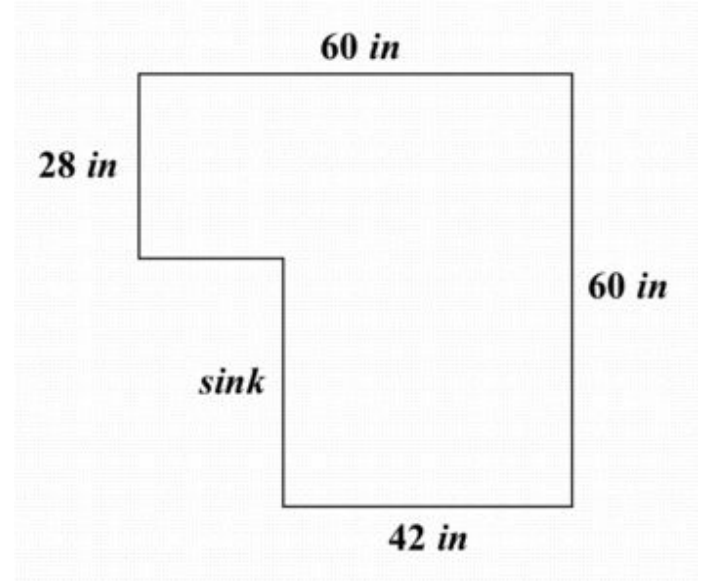
# Teresa's Tiles

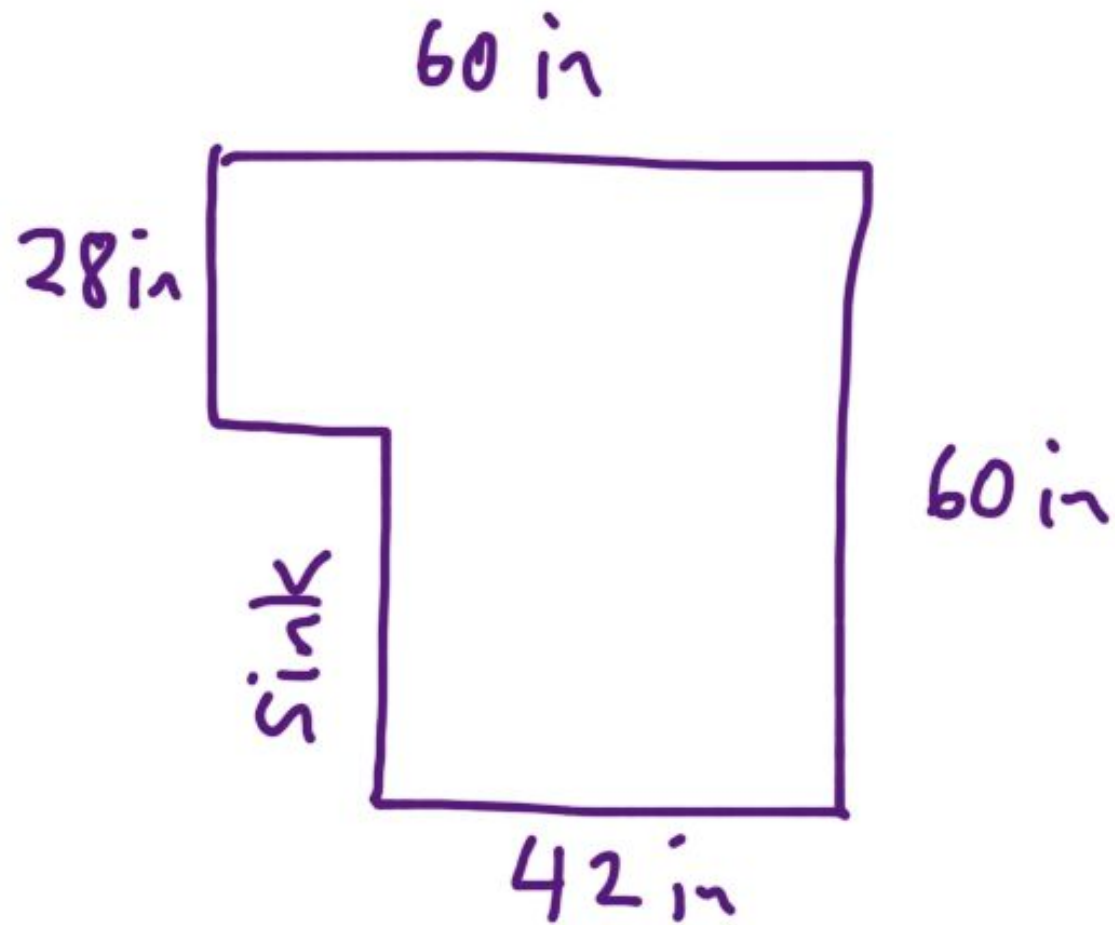
Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.

This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

Questions: How many tiles will she need to buy to cover her floor? How many tiles will she have to cut in order to cover the entire space?

Extra: What is the size, using whole numbers, of the largest square tile that could be used to tile the entire floor with no cut pieces?





# Teresa's Tiles

Things that some “low-performing” 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces

# Sample Grade 3 Test Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

- A. 6     18%
- B. 9     46%
- C. 18    31%
- D. 24    4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

# Sample Test Question Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.



# “Doing Math” or Sense Making?

$$12 - p = 5$$

$$12 - ? = 5$$

[Michelle's son] was struggling to “remember”  $28/4$ .  
When [she] asked him, “How do you think about  $28/4$ ?”  
He replied, “Mom, you aren’t supposed to think about it,  
you are just supposed to do it!!”

# Integrated Math 2 (Grade 10)

|                       | Boy | Girl | Total |
|-----------------------|-----|------|-------|
| Wear Sunscreen        | 84  | 133  | 217   |
| Do Not Wear Sunscreen | 170 | 118  | 288   |
| Total                 | 254 | 251  | 505   |

$P(\text{wears sunscreen})$

$P(\text{is a boy})$

$P(\text{wears sunscreen} \mid \text{is a boy})$

$P(\text{wears sunscreen and is a boy})$

$P(\text{wears sunscreen or is a boy})$

$P(\text{is a boy} \mid \text{wears sunscreen})$

$P(\text{is a boy or a girl})$



# Student Perceptions of Math and Sense Making

1. You aren't supposed to sense-make when doing math.
2. You are supposed to use rules and algorithms and accept whatever answer results.
3. You are supposed to do what your teacher said, even when it doesn't seem like a good idea.

# CCSS Math Practice 1

**Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.

# The Five Strands of Mathematical Proficiency

National Research Council, 2001, *Adding it up: Helping children learn mathematics*.

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

“Productive disposition is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”

# Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

# Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

| I Notice | I Wonder |
|----------|----------|
|          |          |

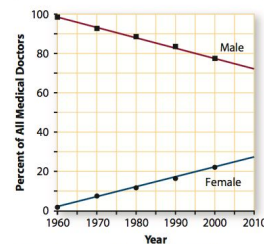
## LESSON 2



# Linear Equations and Inequalities

**F**or most of the twentieth century, the vast majority of American medical doctors were men. However, during the past 40 years there has been a significant increase in the number of women graduating from medical schools. As a result, the percent of doctors who are women has grown steadily to nearly 25% in 2000. The graph on the next page shows this trend.

Male and Female Medical Doctors



Source: [www.ama-assn.org/ama/pub/article/171-195.html](http://www.ama-assn.org/ama/pub/article/171-195.html)

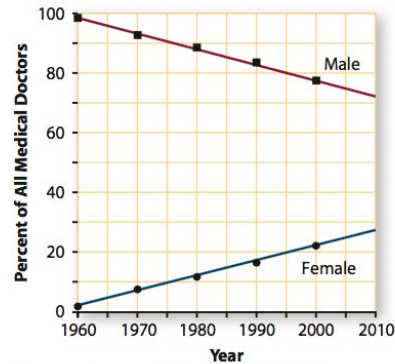
## Think About This Situation

Study the trends in the percentage of male and female medical doctors in the United States between 1960 and 2000.

- How would you describe the trends shown in the data plots and the linear models that have been drawn to match patterns in those points?
- Why do you suppose the percentage of women doctors has been increasing over the past 40 years?
- Would you expect the trend in the graph to continue 10 or 20 years beyond 2000?
- How would you go about finding function rules to model the data trends?
- If you were asked to make a report on future prospects for the percentages of male and female doctors, what kinds of questions could you answer using the linear models?

In this lesson, you will explore ways to express questions about linear functions as equations or inequalities. You will use tables, graphs, and symbolic reasoning to solve those equations and inequalities and to interpret your solutions in problem contexts.

### Male and Female Medical Doctors



Source: [www.ama-assn.org/ama/pub/article/171-195.html](http://www.ama-assn.org/ama/pub/article/171-195.html)

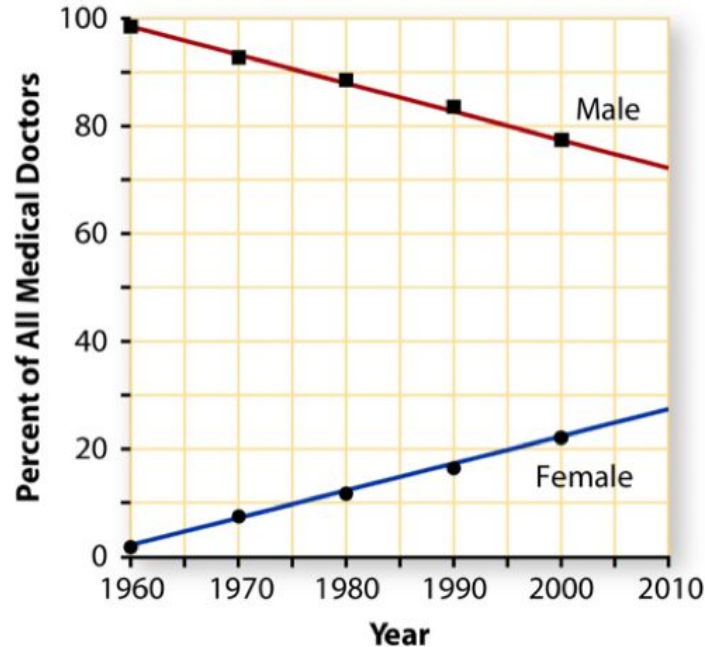
### Think About This Situation

Study the trends in the percentage of male and female medical doctors in the United States between 1960 and 2000.

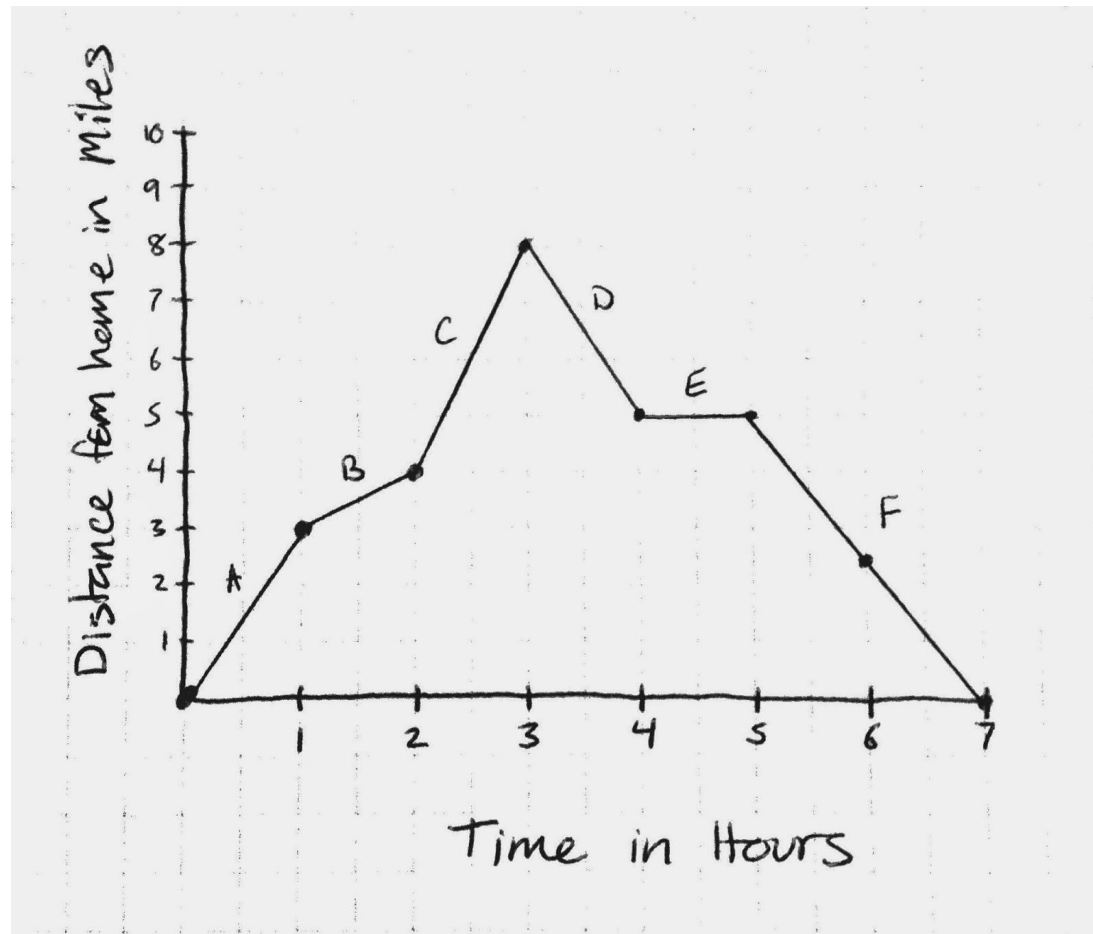
- a** How would you describe the trends shown in the data plots and the linear models that have been drawn to match patterns in those points?
- b** Why do you suppose the percentage of women doctors has been increasing over the past 40 years?
- c** Would you expect the trend in the graph to continue 10 or 20 years beyond 2000?
- d** How would you go about finding function rules to model the data trends?
- e** If you were asked to make a report on future prospects for the percentages of male and female doctors, what kinds of questions could you answer using the linear models?

# Get Rid of the Question

Male and Female Medical Doctors









**Tina Cardone** @crstn85 · Nov 24

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout



**Tina Cardone** @crstn85 · Nov 24

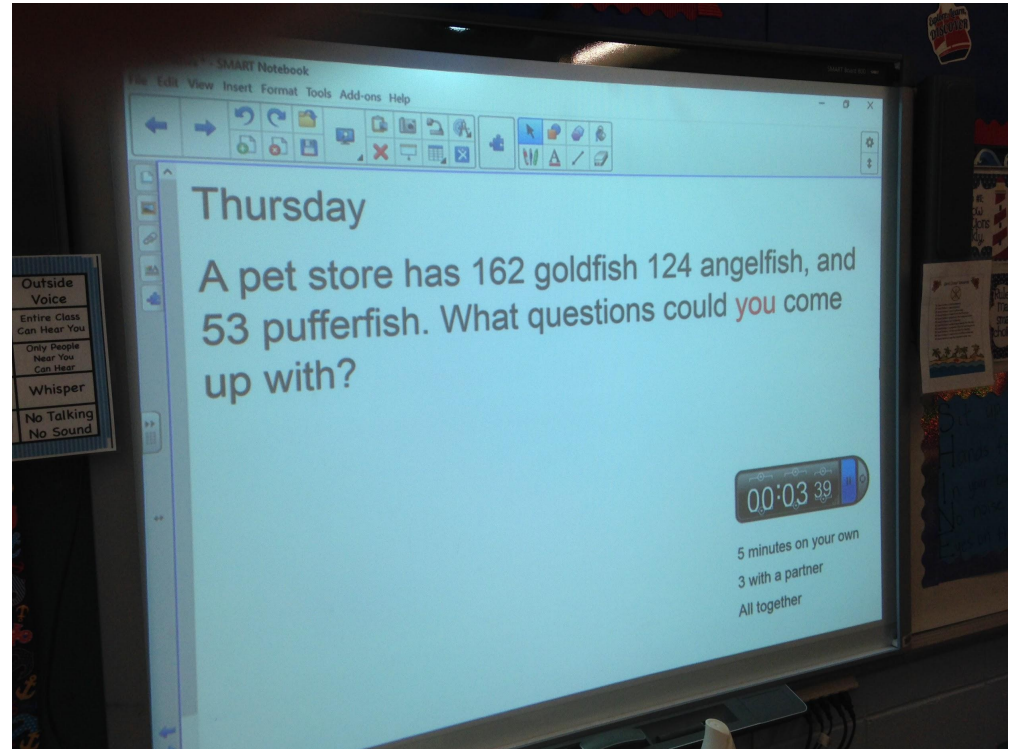
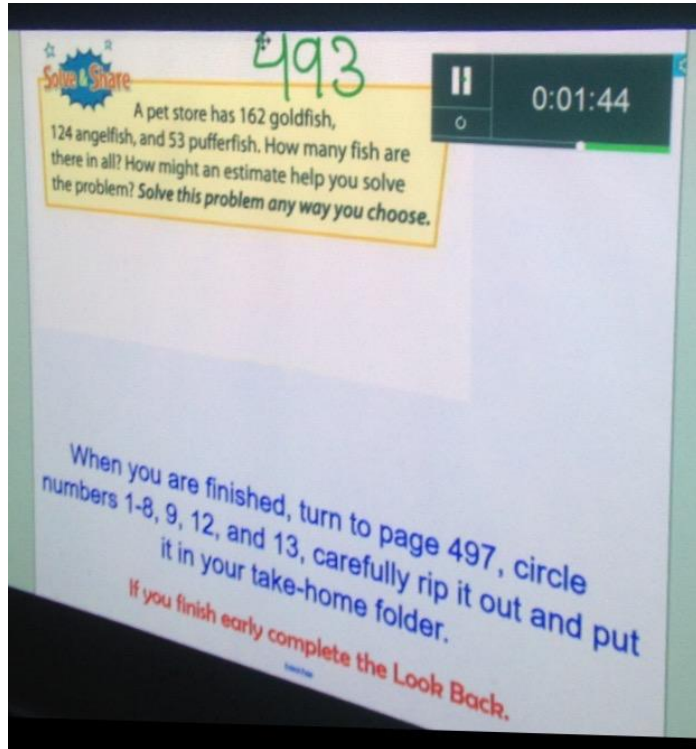
@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)  
[drawingonmath.blogspot.com/2014/11/distan...](http://drawingonmath.blogspot.com/2014/11/distance-graph.html)



<http://drawingonmath.blogspot.com/2014/11/distance-graph.html>

Annie Fetter  
@MFAnnie

# Ask For Questions, Not Answers



pufferfish. What questions  
could you come up with?

How much more does the  
goldfish's has more than  
the angelfish?

What is the  
total of all  
fishes?

How much does  
the pufferfish and the goldfish have altogether?  
How much does the angelfish and

**What could the questions be now?**

how much more is  
blah blah blah than blah  
blah blah?

# Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.

# Get Rid of the Question and the Numbers

Raul had some pet mice. Xavier gave him some more mice.

Raul had some pet mice. Xavier gave him 3 more mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

*A Numberless Word Problem from Brian Bushart, [bstockus.wordpress.com](http://bstockus.wordpress.com)*

**Annie Fetter**  
**@MFAnnie**



Caitlyn is still trying to make brownies for the class. She has the mix and milk but needs to go get eggs. A carton of eggs weighs some 24 pounds. Each carton has 12 eggs. Each carton costs a 1 dollar amount.

How much does one egg weigh? (in ounces)

6

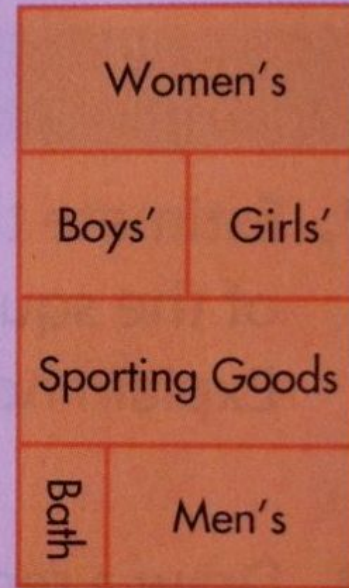
Caitlyn realized that she needs one more ingredient. She forgot the vegetable oil. The oil is sold in 2 liter bottles. She needs a certain number of 6 bottles for the brownies. Each bottle cost a 6.25 dollar amount. Caitlyn brings \$20 with her to the store.

\$5 1p 4c

From Kat Kulis, grade 5, Windham Center School, Windham, CT

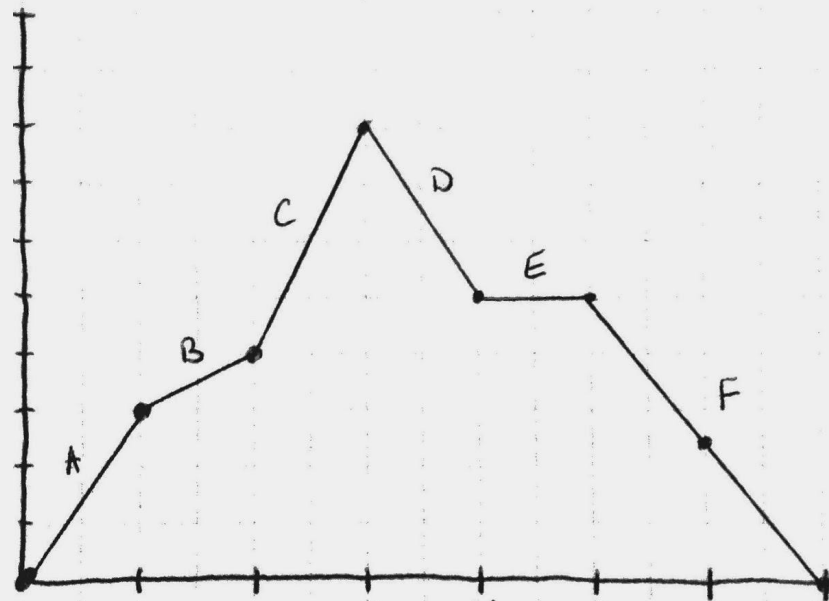
# Get Rid of the Question and the Numbers

A store has the floor plan shown. The area of the women's department is





Distance from home in miles



Time in Hours

# Numbers vs. Relationships

$$A = \pi r^2$$

# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer.

# Give the Answer

## ◆ Math Message Follow-Up

### WHOLE-CLASS ACTIVITY

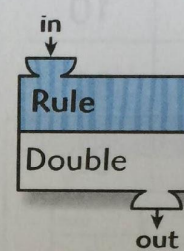
Draw or display a function machine and “What’s My Rule?” table. (See Advance Preparation.)

Ask children to imagine that the **function machine** works like this:

- A number (the **input**) is dropped into the machine,
- the machine changes the number according to a rule,
- and a new number (the **output**) comes out the other end.

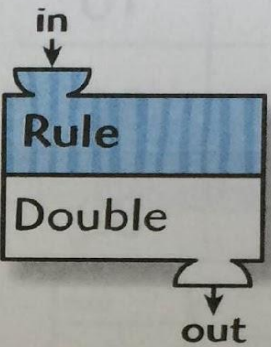
The **rule** for the Math Message problem is “Double the number.” Write the word *Double* in the function machine.

Point out the “**What’s My Rule?**” table. Discuss the 8 in the *in* column and the 16 in the *out* column. Explain to children that numbers in the *in* column represent the numbers of bacteria now. Corresponding numbers in the *out* column represent the numbers of bacteria 20 minutes from now.



| in  | out |
|-----|-----|
| 8   | 16  |
| 50  | 100 |
| 200 | 400 |
| 75  | 150 |
| 150 | 300 |

# Give the Answer



The diagram shows a function block with an input port labeled 'in' at the top and an output port labeled 'out' at the bottom. The block is divided into two sections: 'Rule' (top, blue background) and 'Double' (bottom, white background). The 'Rule' section is highlighted with a blue background.

| in  | out |
|-----|-----|
| 8   | 16  |
| 50  | 100 |
| 200 | 400 |
| 75  | 150 |
| 150 | 300 |

## ► Math Message

You know that  $2.4 * 1 = 2.4$ . Will  $2.4 * 1.8$  be greater than or less than 2.4?

Greater than How do you know? Share your conjecture and argument with a partner. **GMP3.1**

### Math Message 6.8

"Notice and Wonder" - "I notice...."

$$\begin{array}{l} 2.4 * 1 = 2.4 \\ 2.4 * 1.8 = 4.32 \end{array} \left. \vphantom{\begin{array}{l} 2.4 * 1 = 2.4 \\ 2.4 * 1.8 = 4.32 \end{array}} \right\} \begin{array}{l} \text{What do} \\ \text{you} \\ \text{notice?} \end{array}$$

- When you multiply by 1, you get same answer as what you multiplied by.
- They multiplied  $8 \times 4 = 32 \rightarrow$  and they multiplied  $4 \times 4$ .
- $2.4 < 4.32$
- Second problem is wrong.  $\rightarrow$  It's not multiplying, it's adding.
- Both problems have 2.4
- ~~Take out~~ the 3 for swapping 2 and 4 and get same answer as problem 1.
- 4.32 is nearly 2 times as much as 2.4
- Can't multiply  $2 \times 1 = 4$ .

### Math Message 6.8

"Notice and Wonder" - "I notice...."

$$\begin{array}{l} 2.4 * 1 = 2.4 \\ 2.4 * 1.8 = 4.32 \end{array} \left. \vphantom{\begin{array}{l} 2.4 * 1 = 2.4 \\ 2.4 * 1.8 = 4.32 \end{array}} \right\} \begin{array}{l} \text{What do} \\ \text{you} \\ \text{notice?} \end{array}$$

- It's the same decimals.  $\rightarrow 2.4$  is the same as answer.
- When multiplying, the product is less.
- The first problem equals the same number.
- By adding  $\frac{8}{10}$  to 1, the answer was changed to 4.32.
- Almost all the same numbers except  $\frac{8}{10}$ .
- The decimals in the ones place is adding up to 2.
- In the second problem it looks like the answer is incorrect because the whole #s and decimals multiplied would give 2 instead of 4.
- Both problems contain (2) two's and (2) 4's.



# Give *an* Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

- It takes 8 minutes to mix the batter.
- The cookies bake for 9 minutes.
- For 6 minutes they cool.

If the answer is 23 minutes, what is the question?

If the answer is 3 minutes, what is the question?

If the answer is bake, what is the question?

# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

This can be really simple:

“Tell me something about number 7.”

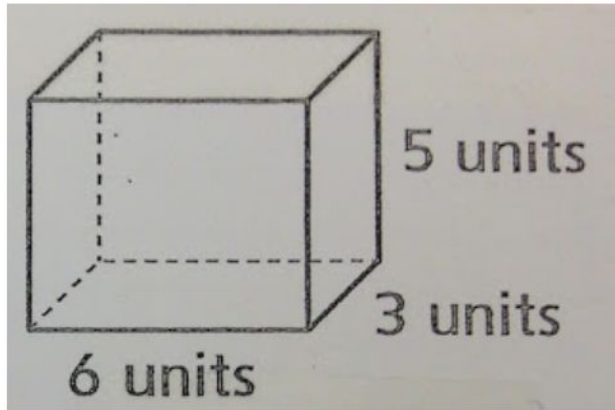
*instead of*

“What's the answer to number 7?”

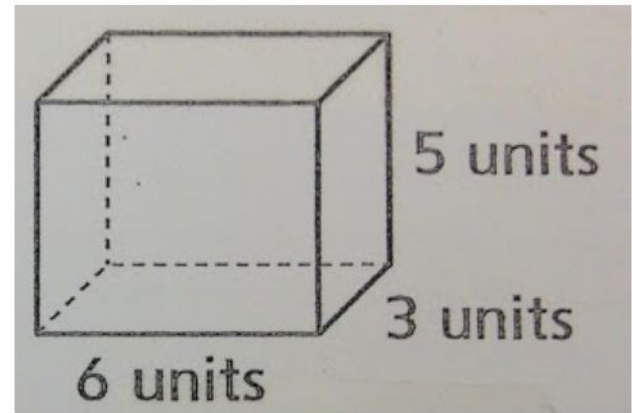


# Ask About Ideas, Not Answers

It can be a little more complex:



*instead of*

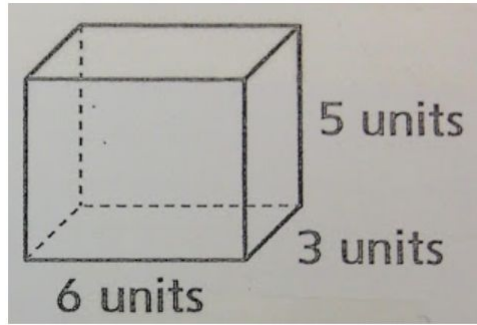


**Tell me everything you can about this figure.**

**Find the volume of the rectangular prism.**

*(from Joe's blog, October 10, 2016)*

There's nothing inside.



Tell me everything you can about this figure.

It's a full cube with a top and everything else.

So if you times what's on the bottom ( $6 \times 3$ ) then you times the height you'll get the answer.

It has 4 sides.

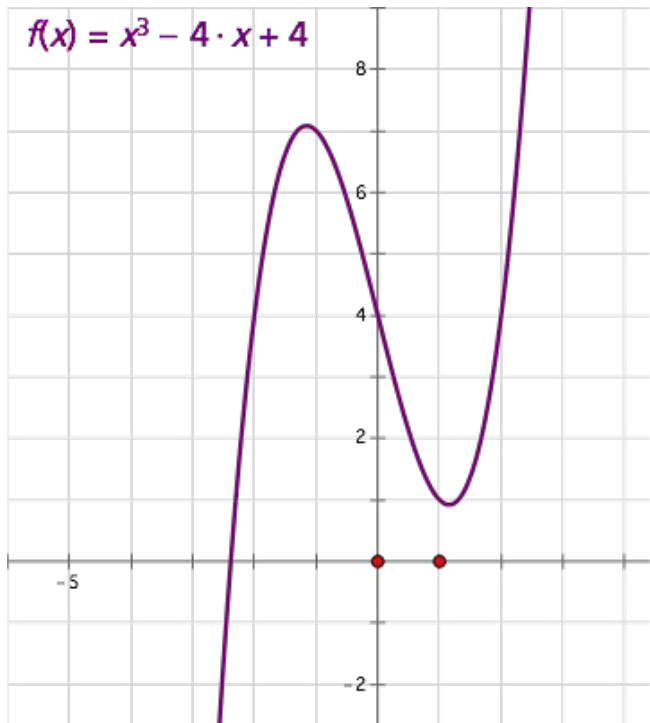
The shape has 12 right (90 deg) angles.

The perimeter is 14 units.

I know this shape is made up of squares.

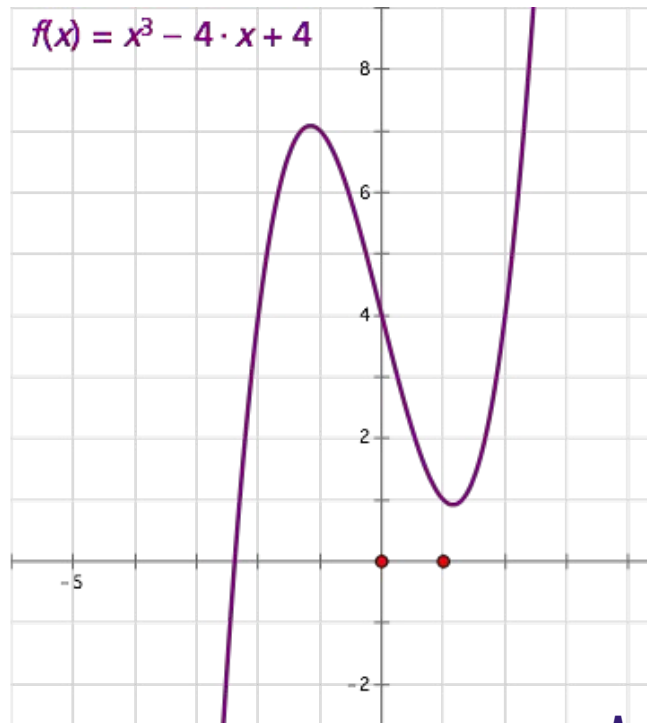
# Ask About Ideas, Not Answers

It can be a little more complex:



Tell me everything you can about the derivative of this function.

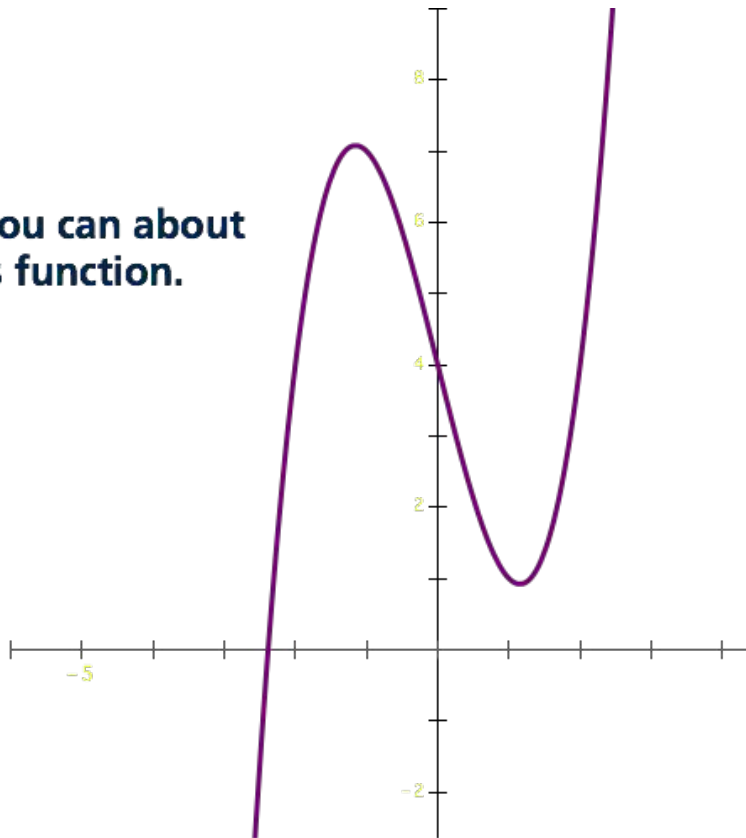
*instead of*



Find the derivative of this function.

# Ask About Ideas, Not Answers

Explain everything you can about the derivative of this function.



# Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”

# “Phone in Pocket”

Are you asking ***idea-focused*** questions or ***answer-focused*** questions? Record yourself and find out!

**#ToVForRatio**

# Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.
- Get rid of the question *and* the numbers.
- Give the answer.
- Ask about ideas, not answers.

# **Moment for Reflection and Personal (Possibly Public) Commitments**



# Thanks!

## Annie

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