

# Making the Shift Towards Sense Making

**Annie Fetter**

anniefetter@gmail.com, @MFAnnie

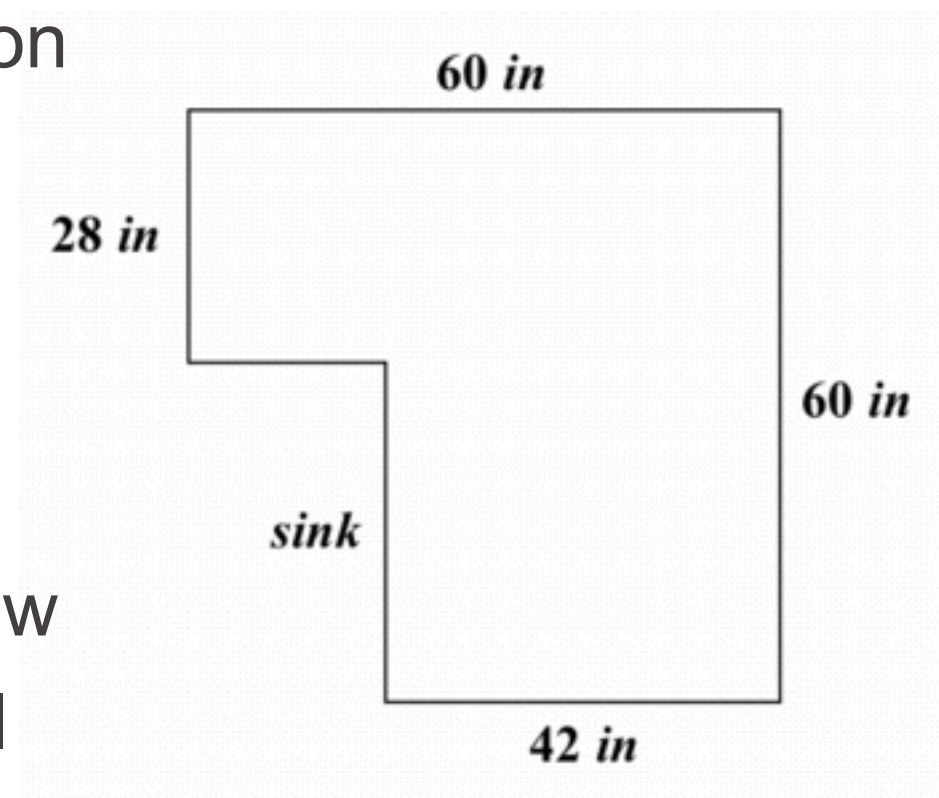
2021 GSDMC Virtual Speaker Series

Slides and links to related resources will be available on my blog after the talk:

[annie.mathematicalthinking.org](https://annie.mathematicalthinking.org)

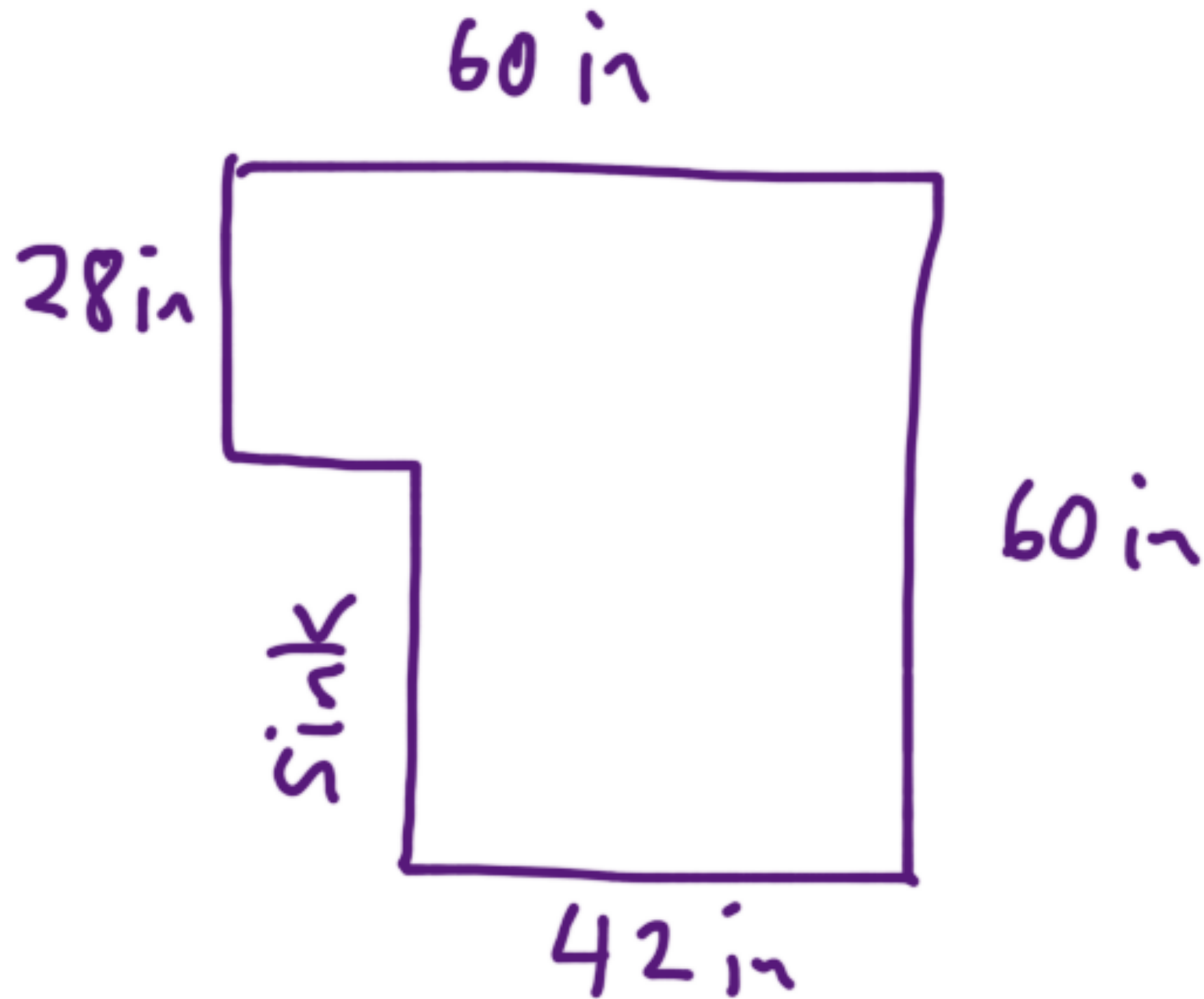
# Teresa's Tiles

Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.



This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

**Questions:** How many tiles will she need to buy to cover her floor?  
How many tiles will she have to cut in order to cover the entire space?



# Teresa's Tiles "Scenario"

Things that some "low-performing" 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces



**There are 125 sheep and  
5 dogs in a flock.  
How old is the shepherd?**

Robert Kaplinsky (@robertkaplinsky) gave this to 32 eighth grade students. How many do you think attempted to find a numerical answer? *(Put your guess in the chat)*

# How Old is the Shepherd?

75% gave numerical answers.

100% of his sixth graders gave numerical answers.

In the original research paper [Reusser 1986],  
“...three out of four school children will  
produce a numerical answer to this problem.”

<http://robertkaplinsky.com/how-old-is-the-shepherd/>  
<https://www.youtube.com/watch?v=kibaFBgaPx4>

# Sample Grade 3 Test Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

A. 6      18%

B. 9      46%

C. 18      31%

D. 24      4%

Combined scores of the 160 third graders in a group of four low-performing schools I used to support.

# Sample Test Question Revised

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.



# CCSS Math Practice 1

**Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.

They monitor and evaluate their progress and change course if necessary.

# Integrated Math 2 (Grade 10)

	Boy	Girl	Total
Wear Sunscreen	84	133	217
Do Not Wear Sunscreen	170	118	288
Total	254	251	505

$P(\text{wears sunscreen})$

$P(\text{is a boy})$

$P(\text{wears sunscreen} \mid \text{is a boy})$

$P(\text{wears sunscreen and is a boy})$

$P(\text{wears sunscreen or is a boy})$

$P(\text{is a boy} \mid \text{wears sunscreen})$

$P(\text{is a boy or a girl})$

# “Doing Math” or Sense Making?

$$12 - p = 5$$

$$12 - ? = 5$$

[Michelle's son] was struggling to “remember”  $28/4$ . When [she] asked him, “How do you think about  $28/4$ ?” He replied, “Mom, you aren’t supposed to think about it, you are just supposed to do it!!”

# Teacher Knows Best

$$\frac{3}{4} + \frac{5}{8} = \frac{8}{12}$$



# Student Perceptions of Math and Sense Making

1. You aren't supposed to sense-make when doing math.
2. You are supposed to use rules and algorithms and accept whatever answer results.
3. You are supposed to do what your teacher said, even when it doesn't seem like a good idea.

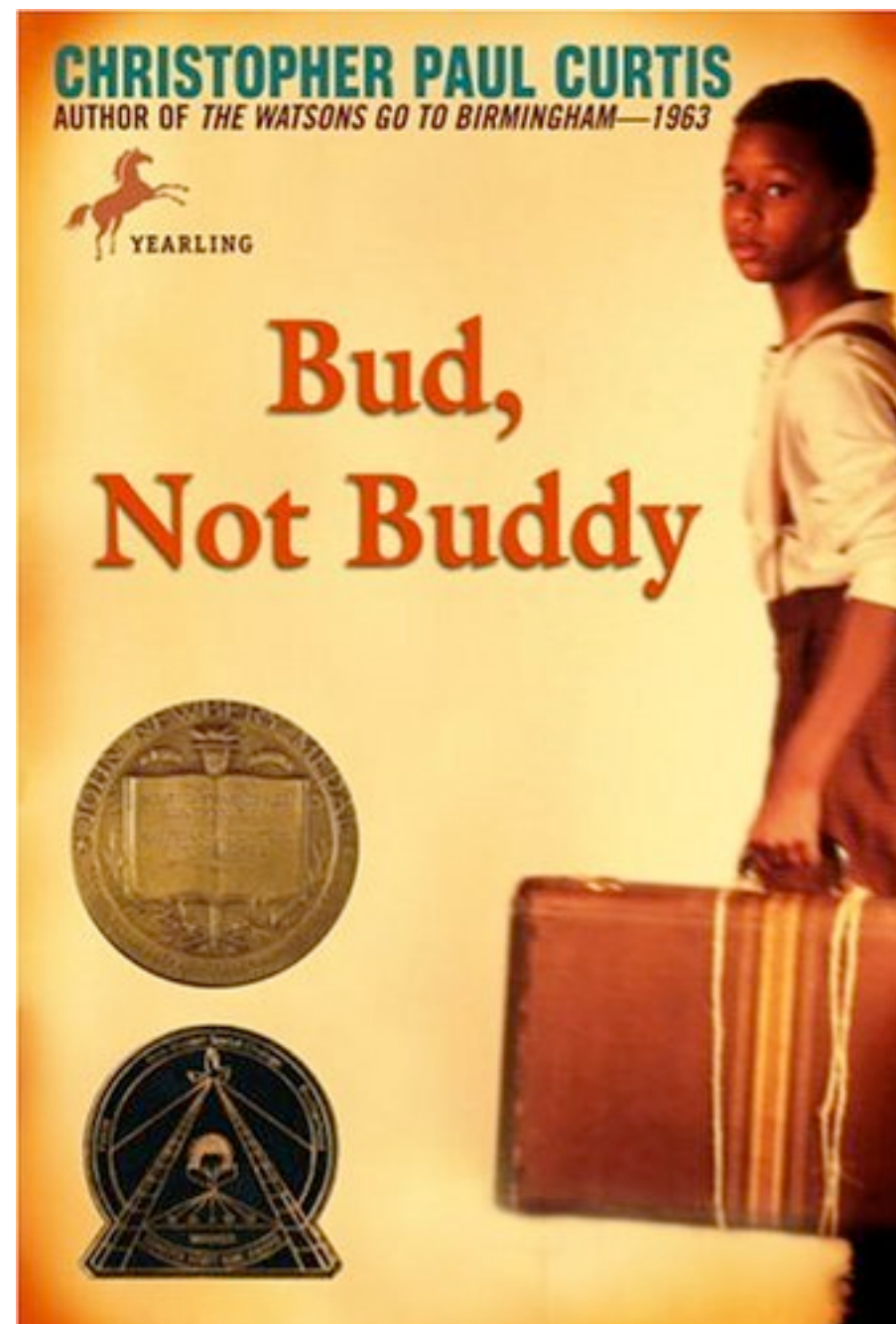
# The Five Strands of Mathematical Proficiency

National Research Council, 2001, *Adding it up: Helping children learn mathematics*.

1. Conceptual understanding
2. Procedural fluency
3. Strategic competence
4. Adaptive reasoning
5. Productive disposition

“Productive disposition is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.”

# Jekyll and Hyde?



**So, what should we do?**

# Encouraging Sense Making

Q: What's one way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question. Literally.

# Get Rid of the Question

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

**I Notice**

**I Wonder**

# Get Rid of the Question

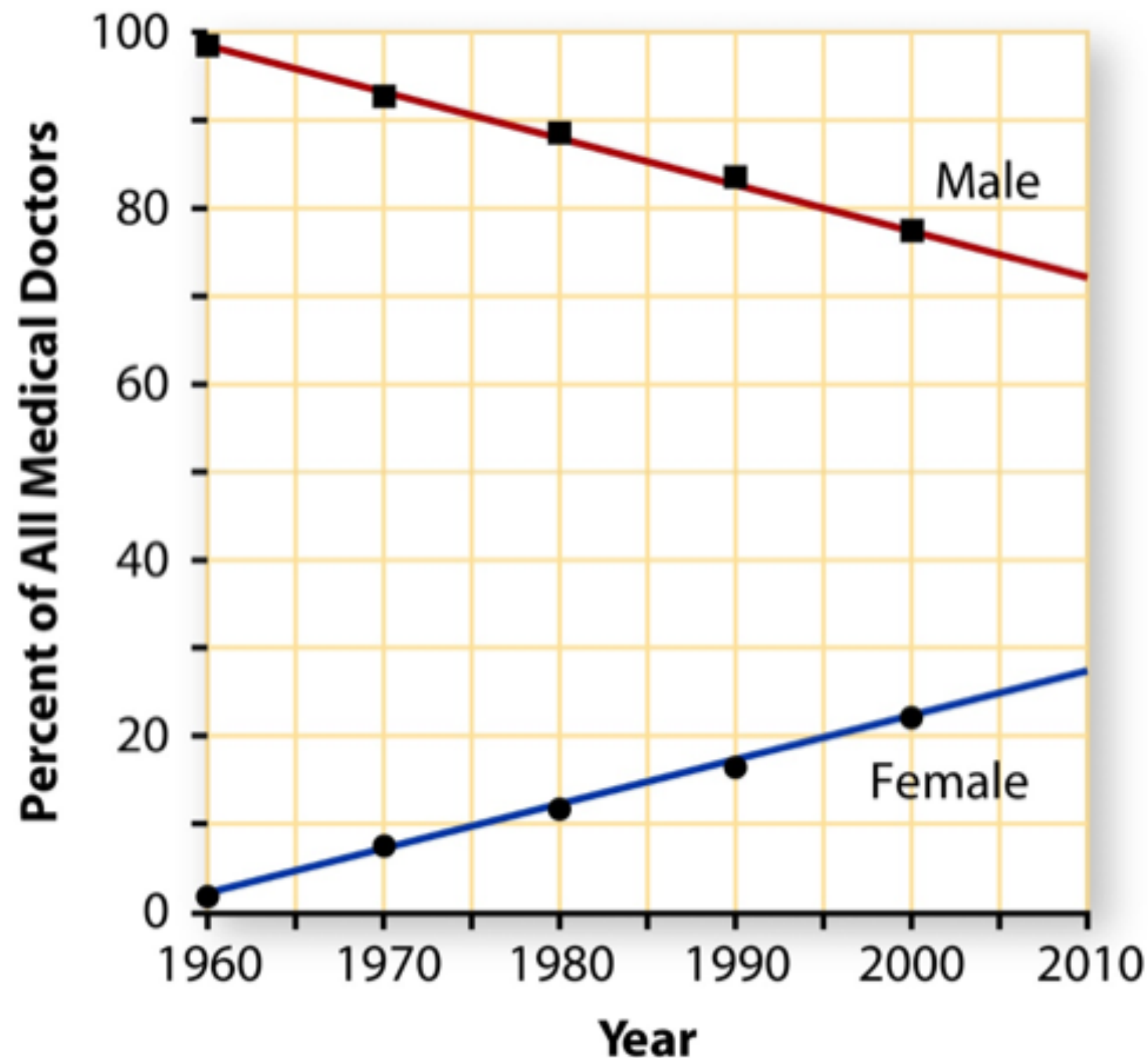
Mr. Gavin has a ladder that is  
100 centimeters tall.

Ms. Cornell has a ladder that  
is 2 meters tall.



# Get Rid of the Question

## Male and Female Medical Doctors

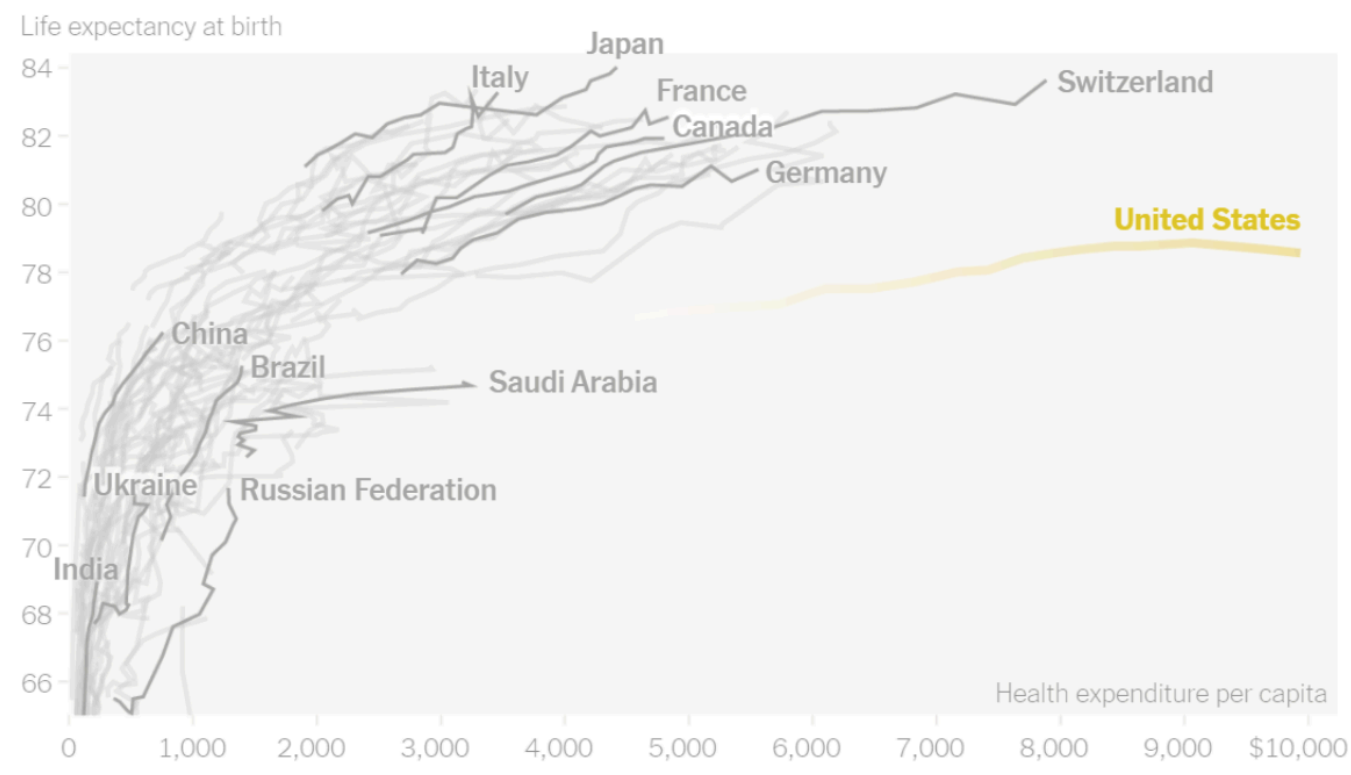




# Get Rid of the Question

## *What's Going On in This Graph? | Global Life Expectancy & Health Expenditures*

How has the relationship between life expectancy and health expenditures by country changed since 2000?



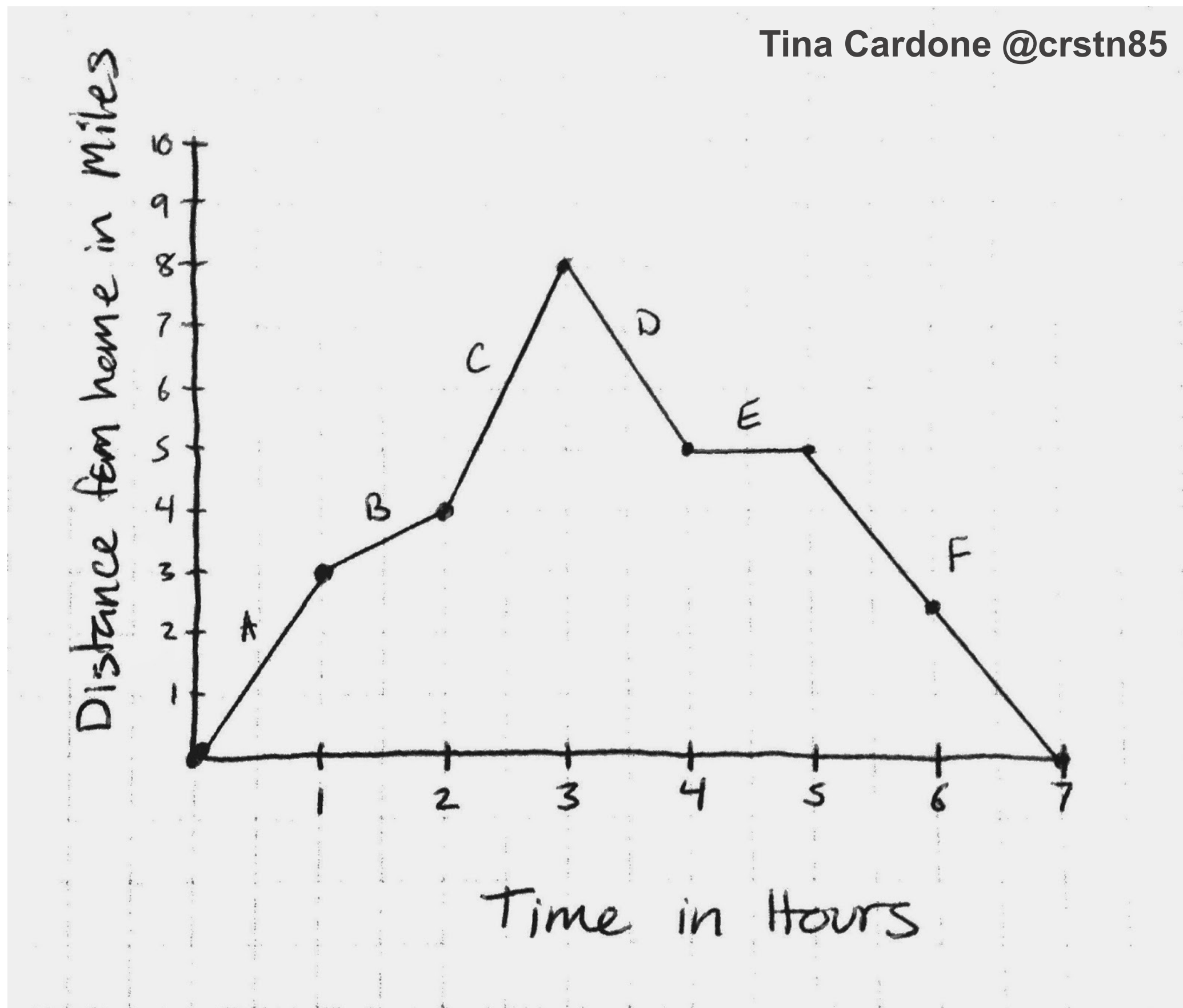
*NY Times Learning Network, #NYTGraphChat*

# Get Rid of the Question

After looking closely at the graph above (or at this [full-size image](#)), answer these four questions:

- What do you notice?
- What do you wonder?
- What impact does this have on you and your community?
- What's going on in this graph? Write a catchy headline that captures the graph's main idea.

Tina Cardone @crstn85





**Tina Cardone** @crstn85 · Nov 24

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout



**Tina Cardone** @crstn85 · Nov 24

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it)  
[drawingonmath.blogspot.com/2014/11/distan...](http://drawingonmath.blogspot.com/2014/11/distance-graph.html)



<http://drawingonmath.blogspot.com/2014/11/distance-graph.html>

# Encouraging Sense-Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Get rid of the question *and* the numbers.



# Get Rid of the Question and the Numbers

Raul had some pet mice. Xavier gave him some more mice.

Raul had some pet mice. Xavier gave him 3 more mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice.

Raul had some pet mice. Xavier gave him 3 more mice. Now Raul has 8 mice. How many mice did Raul have to start with?

*A Numberless Word Problem from Brian Bushart, [bstockus.wordpress.com](https://bstockus.wordpress.com)*

# Get Rid of the Question OR the Numbers

Caitlyn is still trying to make brownies for the class. She has the mix and milk but needs to go get eggs. A carton of eggs weighs some 24 pounds. Each carton has 12 eggs. Each carton costs a 1 dollar amount.

How much does one egg weigh?  
(in ounces)

6

Caitlyn realized that she needs one more ingredient. She forgot the vegetable oil. The oil is sold in 2 liter bottles. She needs a certain number of 6 bottles for the brownies. Each bottle cost a \$6.25 in amount. Caitlyn brings \$20 with her to the store.

\$5 1p 4c

From Kat Kulis, grade 5, Windham Center School, Windham, CT



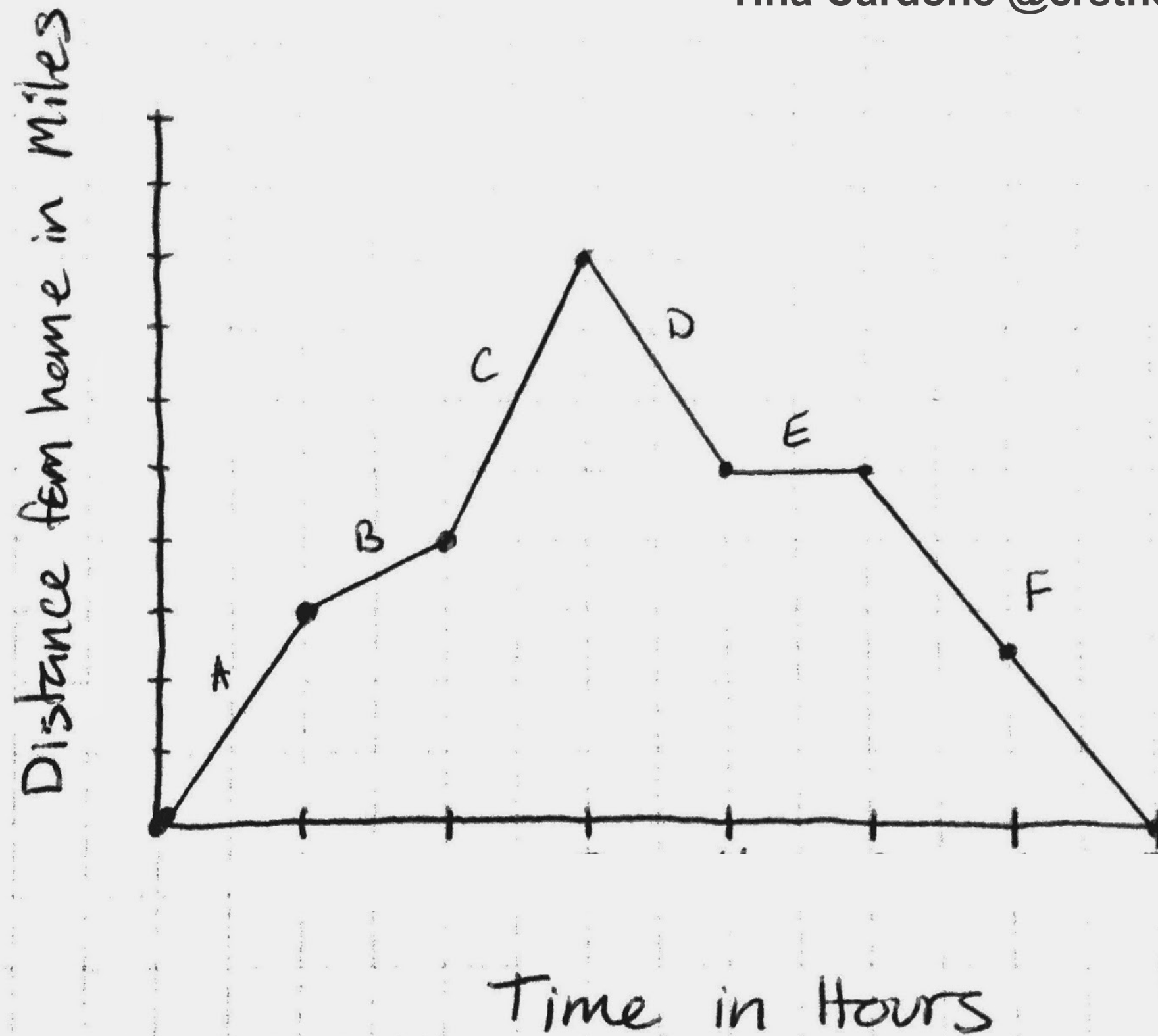
# Get Rid of the Question and the Numbers

A store has the floor plan shown. The area of the women's department is





Tina Cardone @crstn85



# Numbers vs. Relationships

$$A = \pi r^2$$

# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Give the answer.

# Give the Answer (or Several!)

Rachel bakes cookies and delivers them to her friends.

- It takes 8 minutes to mix the batter.
- The cookies bake for 9 minutes.
- For 6 minutes they cool.

**If the answer is 23 minutes, what is the question?**

**If the answer is 3 minutes, what is the question?**

**If the answer is bake, what is the question?**

from Joe Schwartz, @JSchwartz10a

# Encouraging Sense Making

Q: What's another way to cultivate a classroom focused on *sense making* rather than *answer-getting*?

A: Ask about ideas, not answers.

This can be really simple:

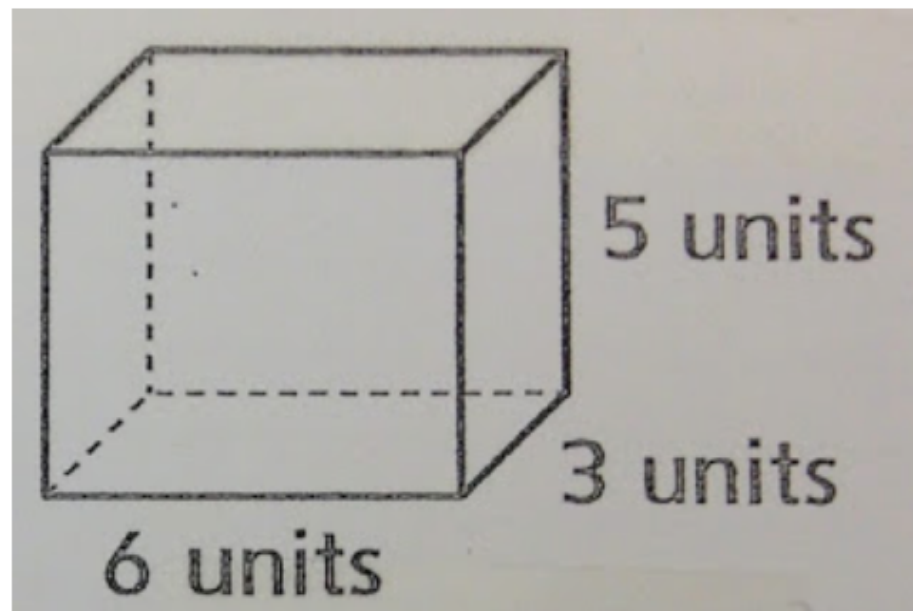
“Tell me something about number 7.”

*instead of*

“What's the answer to number 7?”

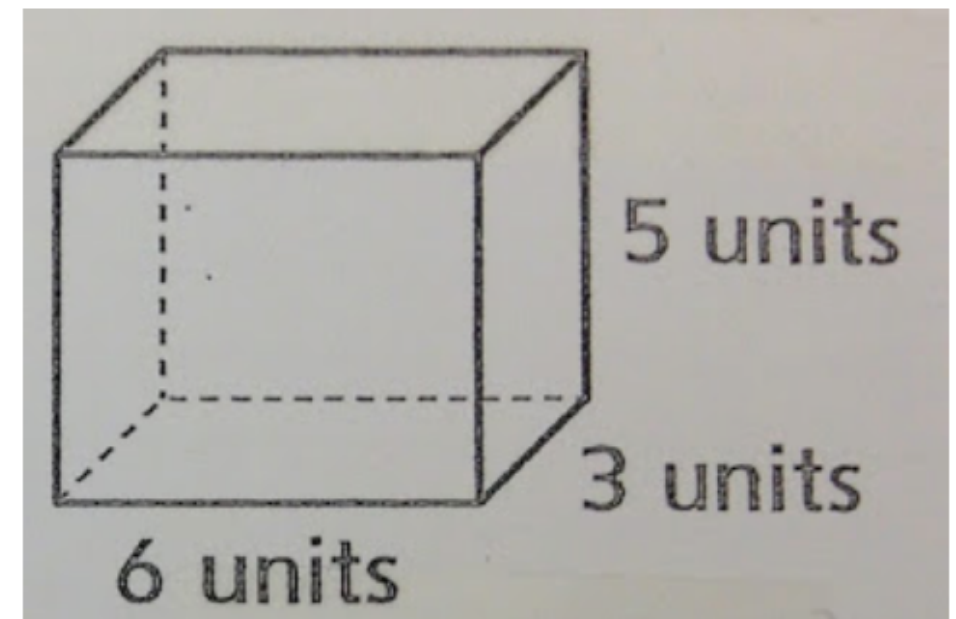
# Ask About Ideas, Not Answers

It can be a little more complex:



**Tell me everything you can about this figure.**

*instead of*



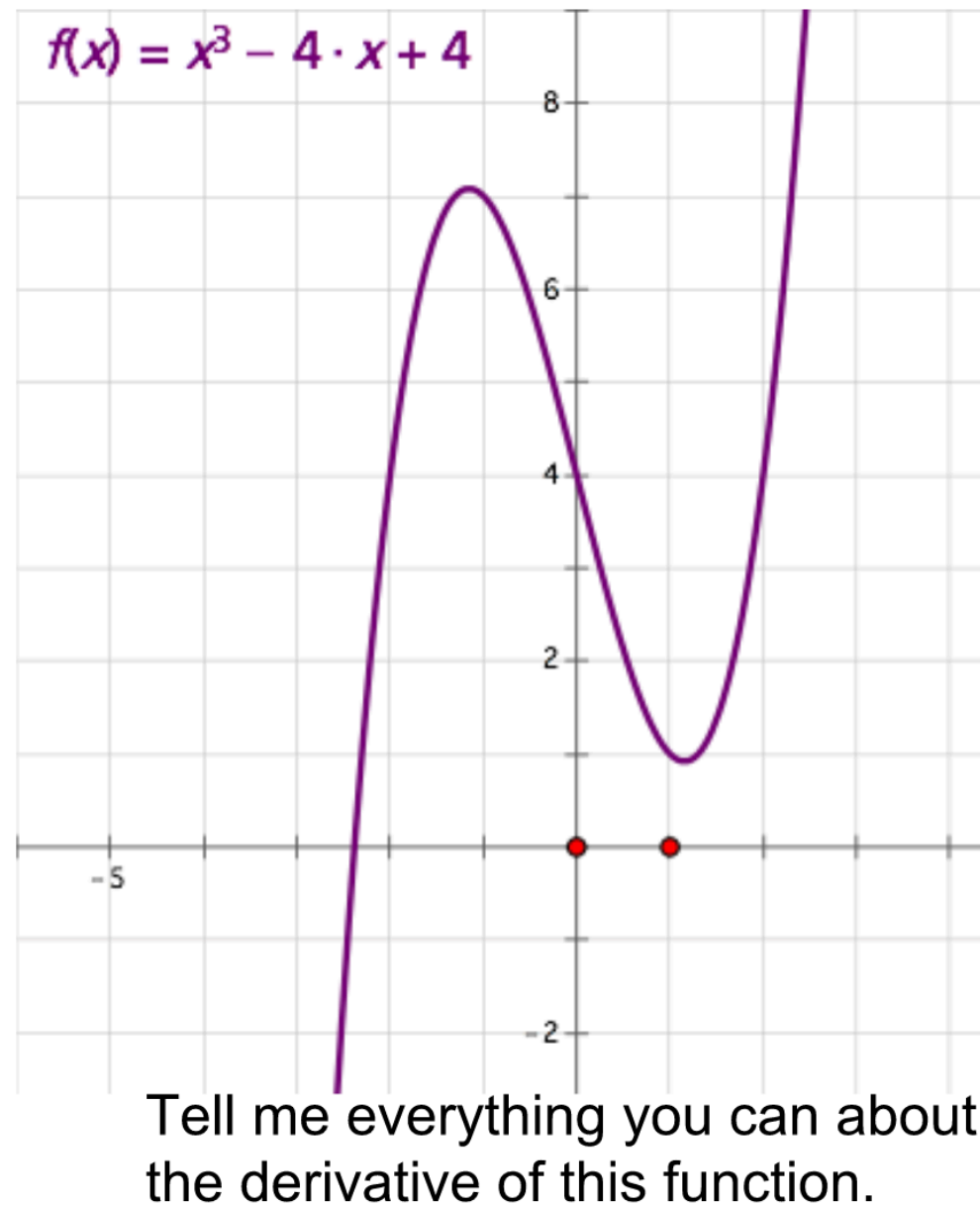
**Find the volume of the rectangular prism.**

*(from Joe Schwartz's blog, [exit10a.blogspot.com](http://exit10a.blogspot.com), October 10, 2016)*

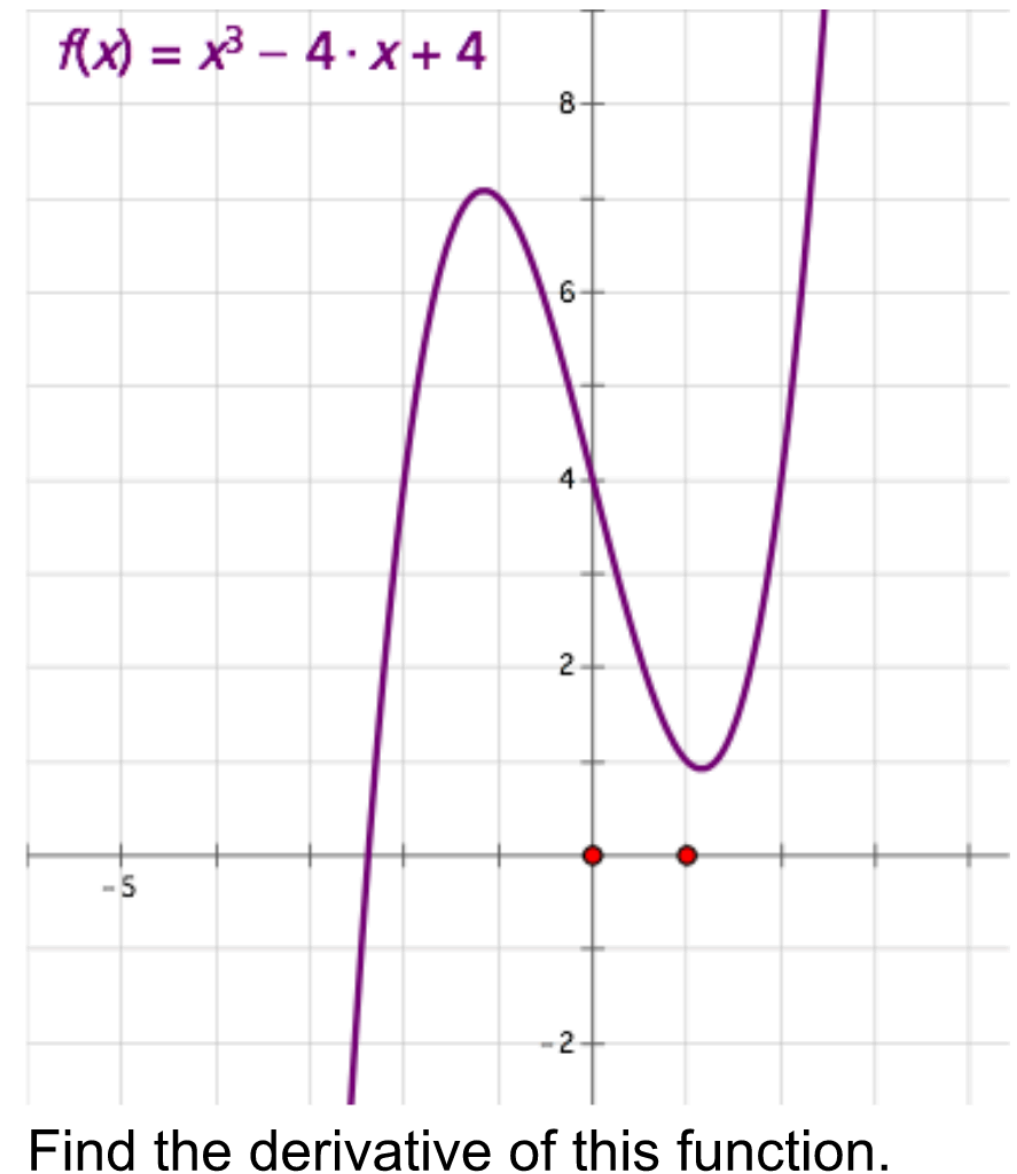


# Ask About Ideas, Not Answers

It can be a little more complex:

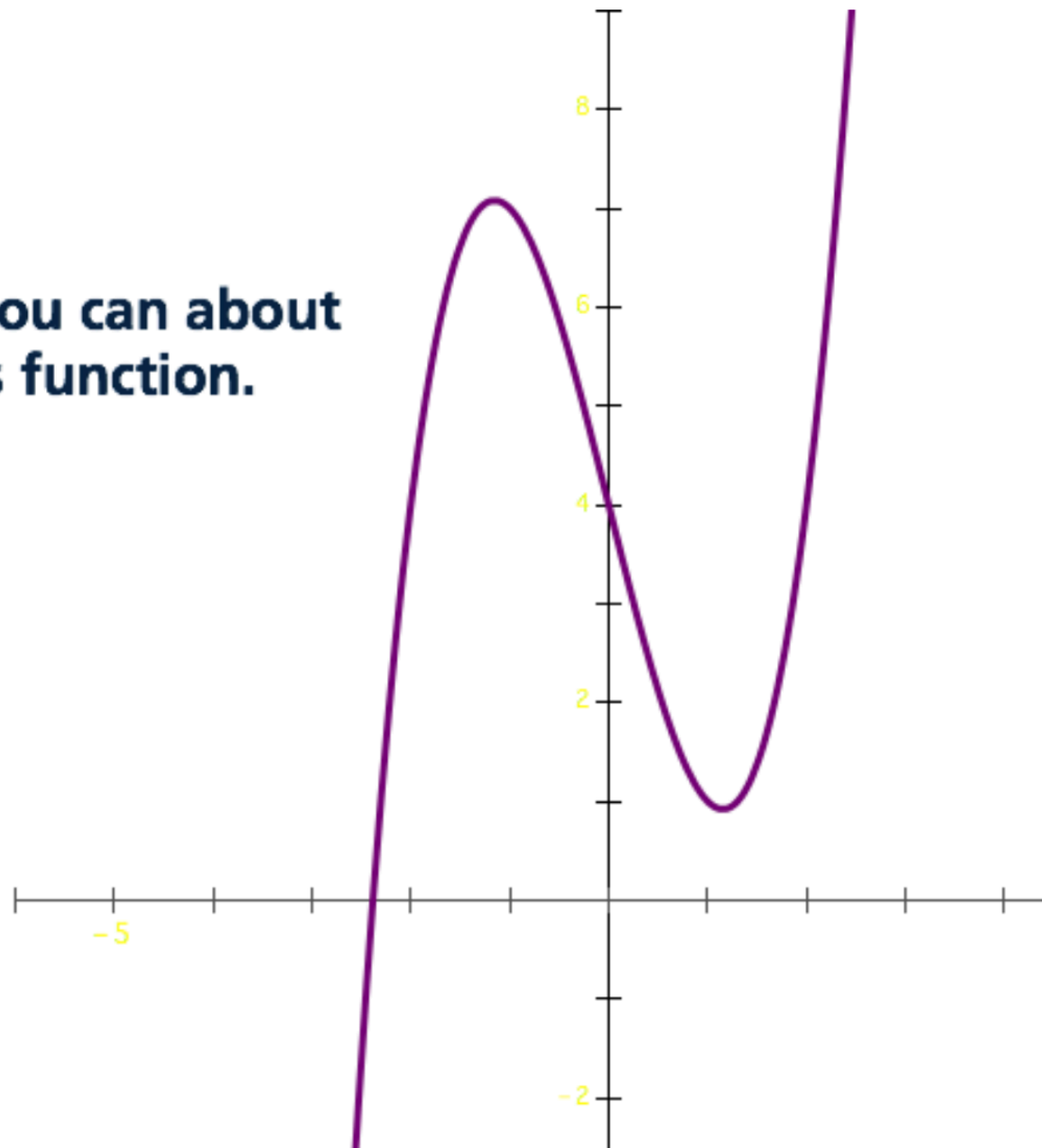


*instead of*



# Ask About Ideas, Not Answers

**Explain everything you can about the derivative of this function.**





# Teacher Questions

“Why?”

“How do you know?”

“How did you decide?”

“Tell me more about that.”

# “Phone in Pocket”

Are you asking *idea-focused* questions or *answer-focused* questions? Record yourself and find out!

**#ToVForRatio**

# Ways to Encourage Sense Making Rather Than Answer Getting

- Get rid of the question.
- Get rid of the question *and* the numbers.
- Give the answer.
- Ask about ideas, not answers.

# **But Wait! There's More!**

# Honoring Students' Ideas

Q: Another?

A: Launch by asking for their ideas instead of telling them things. (Just hush up for a bit!)

# Gathering Ideas as a Launch

## ► Relate Pictures to Tens and Ones

MATH TALK

**MP.1 Make Sense of Problems** Analyze the Problem Discuss the pictures in Exercises 1 and 2. Count the number of cars in the first row. **10 cars** Explain that drivers may be directed to fill a row before parking in the next row of a parking lot. In the same way, people may be asked to fill a row of seats before sitting in the next row at a theater.

- How can a filled row help you count the number of cars or the number of people? **Possible response: A filled row shows ten, so I can use the picture to count tens and extras.**
- How do the cars in Exercise 1 show tens and ones? **2 filled rows show tens and 3 extra cars show ones.**
- How do the people in Exercise 2 show tens and ones? **There are 4 rows of ten with 6 extra ones. This time the ones are at the top and the tens are shown below.**

## ► Math and the Community Theater

Linda and her family go to a show.



1. 10 cars can park in each row.



How many cars are there?

\_\_\_\_\_ tens \_\_\_\_\_ ones = \_\_\_\_\_ cars

2. 10 people can sit in each row.



How many people are there?

\_\_\_\_\_ tens \_\_\_\_\_ ones = \_\_\_\_\_ people

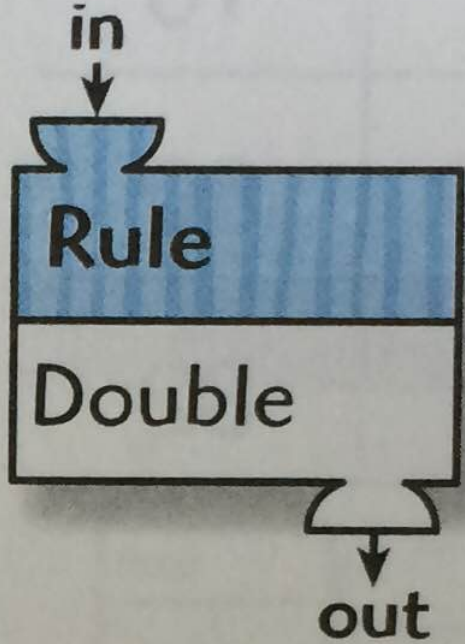
© Houghton Mifflin Harcourt Publishing Company • Image Credits: ©Richard Lewison/Getty Images



# What Do They Notice? Wonder?



# Tell Me Something About This



A diagram of a rule box. It is a rectangle divided into two horizontal sections. The top section is blue with vertical stripes and contains the word "Rule". The bottom section is white and contains the word "Double". An arrow labeled "in" points down into the top of the box. An arrow labeled "out" points down from the bottom of the box.

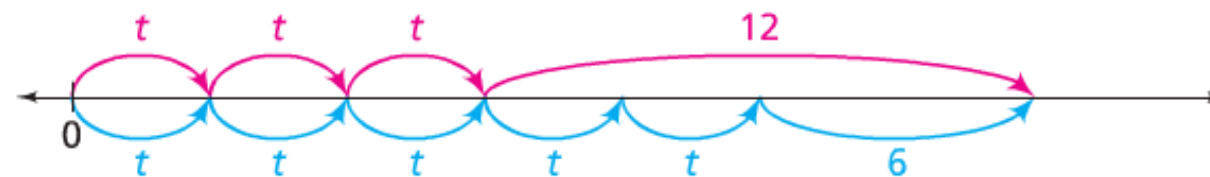
in	out
8	16
50	100
200	400
75	150
150	300



# Stop Talking So Much!

## Equality on the Number Line

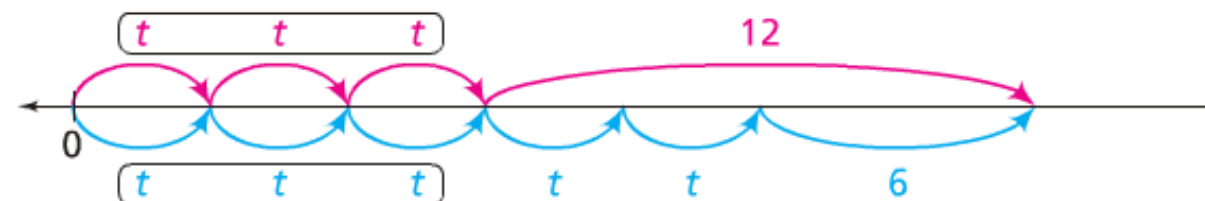
To illustrate the equation  $3t + 12 = 5t + 6$ , you can draw  $t$  as an unknown length. Whatever length you choose for  $t$ , you cannot compare it to the length of 6 or 12, because you do not yet know the value of  $t$ . You do know that every  $t$  has the same length.



The symbols above the number line show  $3t + 12$ . The symbols below the number line show  $5t + 6$ . The equation  $3t + 12 = 5t + 6$  tells you that the two expressions are equal. So, when you draw the two expressions, they can start and end at the same point on the number line.

Look at the  $3t$ 's on the left above and below the line.

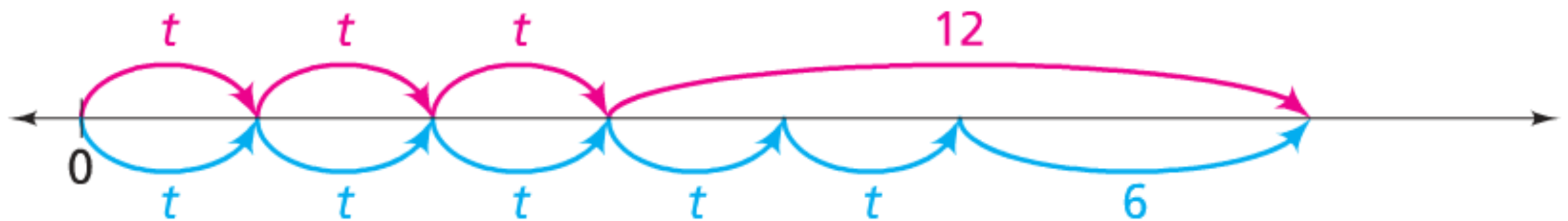
$3t$  is in each expression.



Suppose you ignore the  $3t$ 's on both the top and bottom. The 12 above the line and the  $2t + 6$  below the line start and end at the same point on the number line. So they must be equal.

Ignoring the  $3t$ 's above and below the line is the same as subtracting  $3t$  from both sides of the equation. Above the line, 12 units are left over, and below the line  $2t + 6$  units are left over. Now you have an equation,  $12 = 2t + 6$ , that you can solve using backtracking.

# Let Them Make Sense of Things



# Ways to Honor Students' Ideas

- Get rid of the question.
- Get rid of the question *and/or* the numbers.
- Ask for questions.
- Ask about ideas, not answers.
- Gather their ideas as a launch instead of talking at them.

Reminder #1: Your students all have valid mathematical ideas about pretty much every problem or story.

Your job is to help them believe that.

Reminder #2: In addition to eliciting and honoring students' ideas, your job is to monitor for sense making All. The. Time.

# Let's Reflect

Write down:

What's one thing you *noticed* in these ideas?

What's one thing you're wondering?

# Let's Reflect

Share your ideas in the chat. Format them:

NOTICE: [your idea]

WONDER: [your idea]

# Thank you!

## Annie Fetter

anniefetter@gmail.com, @MFAnnie

Slides and links to related resources will be available on my blog after the talk:

<https://annie.mathematicalthinking.org/>